Maxwell's Electromagnetic equation:

Maxwell's electromagnetic equation are the fundamental equations concerning electromagnetic theory. Following are the four Maxwell's equations in differential form:

1. $\nabla\cdot\vec{D} = \rho$ [Differential form of Gauss's law in electrostatics]

2. $\nabla\cdot\vec{B} = 0$ [Differential form of Gauss's law in magnetostatics]

3. $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ [Differential form of Faraday's law of electromagnetic induction]

4. $\nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$ [Maxwell's modified form of Ampere's circuital law]

where $\vec{D} = \epsilon\vec{E}$ electric displacement vector, ϵ being the permitivity of the medium and $\vec{E}$, the electric field intensity

ρ = electric charge density.

$\vec{B} = \mu\vec{H}$, μ is the magnetic ~~permeabilit~~ permeability & $\vec{H}$, the magnetic field intensity

Maxwell's equations in different media:

(a) Free space (or vacuum)

Here $\rho = 0$ & electrical conductivity $\sigma = 0$,

$\vec{D} = \epsilon_0\vec{E}$ & $\vec{B} = \mu_0\vec{H}$: Here μ_0 & ϵ_0 respectively the permeability & permittivity of free space

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Thus

(1) $\nabla\cdot\vec{D}=0$ & $\nabla\cdot\vec{E}=0$

(2) $\vec{\nabla}\cdot\vec{B}=0$ & $\nabla\cdot\vec{H}=0$

(3) $\nabla\times\vec{E}=0-\mu_0\frac{\partial\vec{H}}{\partial t}$

(4) $\nabla\times\vec{H}=\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

(b) conducting medium:

In conducting medium the positive and negative charges are present in equal amount giving $\rho=0$. Also the displacement current density $J_D\simeq 0$ so

(1) $\nabla\cdot\vec{D}=0$ $(\vec{D}=\epsilon\vec{E})$

(2) $\nabla\cdot\vec{B}=0$ $(\vec{B}=\mu H)$

(3) $\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$

(4) $\nabla\times\vec{H}=\vec{J}$ $(\vec{J}=\sigma E)$

$\mu=$ permeability, $\epsilon=$ permittivity

$\sigma=$ conductivity

(c) Dielectric medium

Here $\rho=0$, $\sigma=0$ so $\vec{J}=\sigma\vec{E}$

1) $\nabla\cdot\vec{D}=0$

2) $\nabla\cdot\vec{B}=0$

3) $\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$

4) $\nabla\times\vec{B}=\frac{\partial\vec{D}}{\partial t}$

(d) Maxwell's equation for static $\vec{E}$ & $\vec{H}$

Here $\frac{\partial \vec{B}}{\partial t} = 0$ & $\frac{\partial \vec{D}}{\partial t} = 0$

1) $\nabla \cdot \vec{D} = 0$

2) $\nabla \cdot \vec{B} = 0$

3) $\nabla \times \vec{E} = 0$

4) $\nabla \times \vec{H} = \vec{J}$

Maxwell's equation in Integral form

① $\oint_s \vec{D} \cdot d\vec{s} = \int_v \rho \, dv$ or $\oint_s \vec{E} \cdot d\vec{s} = q/\epsilon$

② $\oint_s \vec{B} \cdot d\vec{s} = 0$

③ $\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{s}$

④ $\oint H \cdot dl = \int_s \left(\vec{J} + \frac{\partial D}{\partial t} \right) \cdot d\vec{s}$